GRAPH THEORY, JOB-LABOUR SYSTEM AND MANPOWER PLANNING

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Abstract: People working in a community can be grouped into clusters of varying education/training background. Flowing of workers between groups is evident, with movement starting from birth and ending to death. Graph theory is applied to denote the system. Analogy to an electric circuit is referred for quick over-viewing and solution. Figures for manpower planning are predicted.

INTRODUCTION

The administration of a community has the commitment to plan the education and training for providing quality workers to various positions in government and business organisations. The manpower resources of a community can be treated as a system. This system has certain behaviours and can be represented as a "directed graph". With application of graph theory, solution to the flowing between groups of personnels can be found. Based on the solution, manpower planning of a community would be readily handled.

1. Grouping of Manpower

Manpower of a community can be divided into the following groups:

(A) Child (age 14 and below), they are non-workers.

(B) Labour (with threshold of education).

(C) Operator, or Machine Operators (with threshold of education and training)

(D) Craftsman (Operators plus three years of craft training)

(E) Clerk (with threshold of education plus three years of Senior Middle School)

(F) Technician (completed Senior Middle School and four years Tertiary Education), or Associate Professional as it is sometimes called.

(G) Professional (completed Tertiary Education and obtained Professional Qualification)

(H) Manager (with experience of work and ability to manage people for achieving goals)

(I) Retired

2. A Hypothetical Vocational Education System for a Community

It is assumed that a community has a system to teach people to attain the required education and

training for the need of the community. We may call it “Education and Training System”. This system may include the following:

(a) Everyone in the community is to have a nine-year-long compulsory education. This is called “threshold of education”.

(b) People with “threshold of education” are grouped in (b). These people can work as “elementary workers” or “labours”.

(c) Those attained “threshold of education” may go through a three-year-long “Senior Middle School Education” to achieve the qualification of a “clerk”. A “clerk” is grouped in (c).

(d) Any “clerk” may go through a four year long “Tertiary Education” to achieve the qualification of a “technician”. A “technician” is grouped in (d).

(e) Technicians can gain professional experience to become “professionals”. A “professional” is grouped in (e).

(f) Any “labour” after a specific training of one tenth of a year can be employed as “operator” (“machine operator”). An “operator” is grouped in (f).

(g) Any “operator”, after a craft training of four years, can be employed as “craftsmen”. A “craftsman” is grouped in (g).

(h) Everyone, of different VE classes, i.e., (b)…(g), can gain experience and managerial ability. They then become “managers”. A “manager” is grouped in (h).

(i) Any “labour”, “operator”, or “craftsman”, after acquiring the level of “Senior Middle School Education” can be employed as a "clerk".

(j) After attaining the age of 65, everyone in the community shall leave his job and retire. These people are grouped as “retired”, i.e., group (i).

3. Application of Graph Theory

As suggested in para.2, there are movements of the manpower between groups. With the introduction of a “child” group and the omission of the “retired” group, a graph showing the “simplified manpower system” with flowing of human resources is drawn in Fig.1.

It is noted that the graph has nodes (vortex) representing each manpower resource group, whilst the edges (arcs, or links) are used to represent the flow of manpower from one group to another. The edges are arrows, i.e., “directed edges”, and the graph is a “directed graph” or “digraph”.

4. Assumptions of the Simplified Manpower System

The system mentioned in para. 3 has the following assumptions:

(1) Annual birth equals to annual death. The population would therefore remain a constant.

(2) The number of both “annual birth” and “annual death” equals to a fraction of the total working population, i.e., no death is assumed for the group of “Child”.

(3) No “retired” in the system.

(4) Number of people in each group, i.e., A, B, C, D, E, F, G and H are maintained the same.

(5) Death rate in each of the “working people” groups are the same.

(6) All “child” always attain “threshold of education” at the end of age 14.

(7) It takes a “labour” three years to attain the education level of a “clerk”. Same period is required for an “operator” or a “craftsman” to become a “clerk”.

(8) It takes a “clerk” 4 years to attain the education level of a “technician”.

(9) It takes a “labour” 0.1 year to attain the skill level of an “operator”.

(10) It takes an “operator” 4 years to attain the skill level of an “craftsman”.

(11) It takes a “technician” 5 years to become a “professional”.

(12) For every member of other “worker” groups to become a “manager”, two years of job experience is required.

5. Characteristics of the Simplified System

Let A, B, C, D, E, F, G, H be the number of people in each group, and r be the birth rate with the Working Population as the base. Then the following equation are established:

As "child" is for the population of age 14 and below,

$A = 14\, r\, (B + C + D + E + F + G + H)$

Number of births annually = Number of deaths in a year = $r\, (B + C + D + E + F + G + H)$

Deaths in respective Working Groups are then rB, rC, rD, rE, rF, rG, rH respectively.

In order to maintain the population the same in each group, flow from one group to another are thus required.

The following equations show the relationship of IN and OUT at each knot.

$I0a = Iab$

$Iab = Ibc + Ibf + Ibh + Ibx$

$Ibc + Ifc + Igc = Icd + Ich + Icx$

$Icd = Ide + Idh + Idx$

$Ide = Ieh + Iex$

$Ibf = Ifc + Ifg + Ifh + Ifx$

$Ifg = Igc + Igh + Igx$

$Ibh + Ich + Idh + Ieh + Ifh + Igh = Ihx$

Assuming the following

Working Population = 3000000

death rate per "WorkingPopulation" = 0.03

Ibh : Ich : Idh : Ieh : Ifh : Igh = 4 : 5 : 8 : 10 : 5 : 8

Ibc : Ifc : Igc = 8 : 1 : 1

A solution is obtained with flows: ab=90000, bc=42192, bf = 29088, bh = 720, bx = 18000, fc = 5274, gc = 5274, cd = 24840, ch = 900, cx = 27000, de = 7200, dh = 1440, dx = 16200, eh = 1800, ex = 5400, fg = 15714, fh = 900, fx = 7200, gh = 1440, gx = 9000 and hx = 7200.

6. Directed Graph and Relevant Matrices

The “directed graph” of Fig.1 can be expressed with a number of matrices, i.e., adjacent matrix, weighted adjacent matrix, cut matrix, circuit matrix, etc. Algorithms derived from graph theory can be used to compute the solution using computer softwares.

7. Adjacence Matrix

The "adjacence matrix" is a matrix, with a digit "1" to show the movement from one vertex (on the left column) to another (on the top row), and "0" for none. Fig.2 shows the Adjacence Matrix for the simplified system.

8. Weighted Adjacence Matrix

The "flow rate" value from one group to another is "marked" on the edge. This figure is to replace "1" in the "adjacence matrix". This new matrix is the Weighted Adjacence Matrix of the "directed graph". Fig.3 is the "weighted adjacence matrix" of the simplified system.

9. Planning of Education/Training for Flowing Manpower

As indicated in para.2, 4 and 5, education/training should be needed for the following groups of "Working People" before they are appointed to work on the specific jobs:

(a) Senior Secondary Education, expecting 52740 qualified people to fill the jobs.

(b) Tertiary Education, expecting 24840 qualified people to fill the jobs.

(c) Training for Professionals, expecting 7200 qualified people to fill the jobs.

(d) Training for Managers, expecting 7200 qualified people to fill the jobs.

(e) Training for Operators, expecting 29088 qualified people to fill the jobs.

(f) Training for Craftsmen, expecting 15714 qualified people to fill the jobs.

(g) Lifelong education program for Operators and Craftsman, totaling 10548, to attain the standard of Senior Secondary Education.

(h) Lifelong education program for Operators and Craftsman, totaling 2109 (assuming 20% would require), to attain the standard of Tertiary Education.

10. Analogy to an Electric Circuit

The graph of Fig.1 can also be treated as a circuit diagram with the total current equivalent to the number of births per year. The flows in number per year from and to various groups can be regarded as currents. Arrows between "nodes" can be assigned resistance corresponding to the flow rates. The "birth node" is regarded as the electric source, whilst "death" (died) is earthed to complete the circuit. A circuit diagram shown in Fig.4 is a typical representation of the "Simplified System". With the parameters assigned, Kirchhoff's Current Law and Voltage Law can be applied to determine the flow between groups.

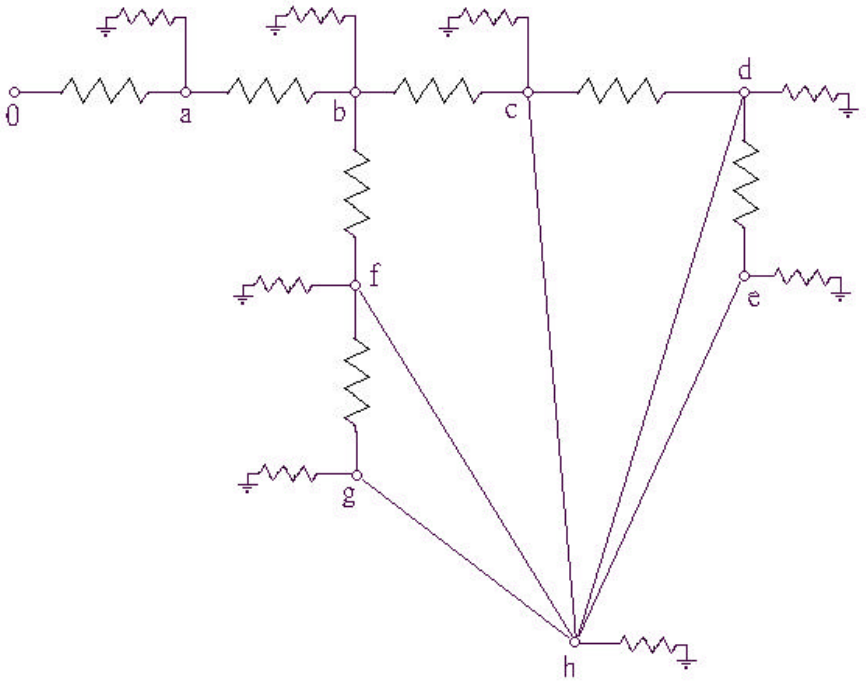

Fig.4 Electric Circuit Representation of the Simplified System

11. Application of Directed Graph Concept to Community Manpower Planning

Manpower planning of a Community requires the anticipation of demand in various group of employees. Use of Directed Graph approach can give an overview of the manpower requirement when statistics of the seven groups are available. Subsystem within a group, e.g., labour for construction industry or for other industries, can be established when needed. Such system approach would give more realistic figures than those derived from Manpower Projection Method which projects figures from historical data of particular group of personnels.

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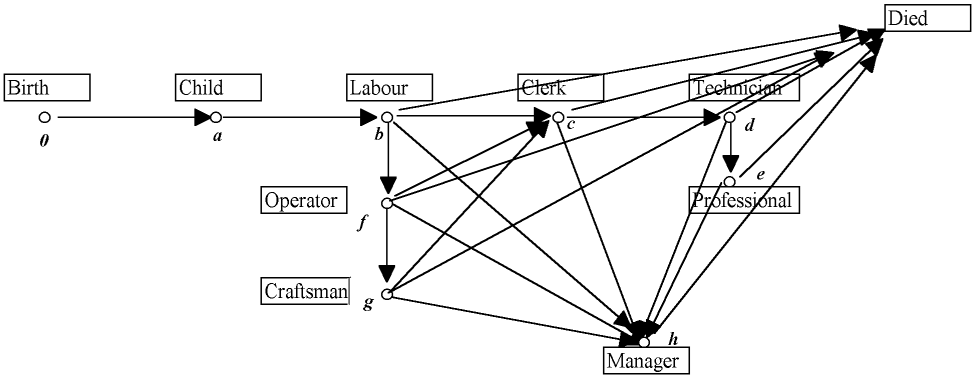

Fig1. Movement of Human Resources in a Community

		0	a	b	c	d	e	f	g	h	x
Birth	0		1								
Child	a			1							
Labour	b				1			1		1	1
Clerk	c					1				1	1
Technician	d						1			1	1
Professional	e									1	1
Operator	f				1				1	1	1
Craftsman	g				1			1		1	1
Manager	h										1
Died	x										

Fig.2 Adjacence Matrix of the Directed Gragh representing the Simplified Manpower System

		a	b	c	d	e	f	g	h	x
Birth	0	90000								
Child	a									
Labour	b			42192			29088		720	18000
Clerk	c				24840				900	27000
Technician	d					7200			1440	16200
Professional	e								1800	5400
Operator	f			5274				15714	900	7200
Craftsman	g			5274					1440	9000
Manager	h									7200
Died	x									

Fig.3 Weighted Adjacence Matrix with "flow" figures as the Weights